Proof of the consistency theorem

Proof of the consistency theorem for the "Follow me" case study.

theorem *consistencyStructurel*
$\exists Structural\_Style' \bullet NewArchitecture2$

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use *axiom$12*

$m2 . load = 8 \Rightarrow (\exists Structural\_Style' \bullet NewArchitecture2)$

use *axiom$8*

$maxLatency = 5 \land m2 . load = 8 \Rightarrow (\exists Structural\_Style' \bullet NewArchitecture2)$

use *axiom$11*

$m1 . load = 2 \land maxLatency = 5 \land m2 . load = 8 \Rightarrow (\exists Structural\_Style' \bullet NewArchitecture2)$

use *axiom$7*

$maxLoad = 5 \land m1 . load = 2 \land maxLatency = 5 \land m2 . load = 8 \Rightarrow (\exists Structural\_Style' \bullet NewArchitecture2)$

use *axiom$4*

$m1 \neq m2$
$\land m1 \neq m1'$
$\land m2 \neq m1'$
$\land maxLoad = 5$
$\land m1 . load = 2$
$\land maxLatency = 5$
$\land m2 . load = 8$
$\Rightarrow (\exists Structural\_Style' \bullet NewArchitecture2)$

use *axiom$19*

$latency (m2, m1) = 2$
$\land m1 \neq m2$
$\land m1 \neq m1'$
$\land m2 \neq m1'$
$\land maxLoad = 5$
$\land m1 . load = 2$
$\land maxLatency = 5$
$\land m2 . load = 8$
$\Rightarrow (\exists Structural\_Style' \bullet NewArchitecture2)$

use *axiom$14*

$replicate\ m2 = m1'$
$\land latency (m2, m1) = 2$
$\land m1 \neq m2$
$\land m1 \neq m1'$
$\land m2 \neq m1'$
$\land maxLoad = 5$
$\land m1 . load = 2$

$\wedge$ *maxLatency = 5*
$\wedge$ *m2 . load = 8*
$\Rightarrow$ *(∃Structural_Style' • NewArchitecture2)*

use *axiom$16*

latency (m1, m2) = 7
$\wedge$ *replicate m2 = m1'*
$\wedge$ *latency (m2, m1) = 2*
$\wedge$ *m1* ≠ *m2*
$\wedge$ *m1* ≠ *m1'*
$\wedge$ *m2* ≠ *m1'*
$\wedge$ *maxLoad = 5*
$\wedge$ *m1 . load = 2*
$\wedge$ *maxLatency = 5*
$\wedge$ *m2 . load = 8*
$\Rightarrow$ *(∃Structural_Style' • NewArchitecture2)*

prove by reduce

latency (m1, m2) = 7
$\wedge$ *replicate m2 = m1'*
$\wedge$ *latency (m2, m1) = 2*
$\wedge$ ¬ *m1 = m1'*
$\wedge$ ¬ *m2 = m1'*
$\wedge$ *maxLoad = 5*
$\wedge$ *m1 . load = 2*
$\wedge$ *maxLatency = 5*
$\wedge$ *m2 . load = 8*
$\Rightarrow$ *{m1} ∪ ({m1} ∪ ({m1'} ∪ {m2})) = {m1} ∪ ({m1'} ∪ {m2})*
$\wedge$ *(¬ x__1 = y__1*
$\wedge$ *(x__1 = m1 ∨ x__1 = m1' ∨ x__1 = m2)*
$\wedge$ *(y__1 = m1 ∨ y__1 = m1' ∨ y__1 = m2)*
$\Rightarrow$ *(∃T:* seq *Manager*
• *(∀i:* ℕ | *1 ⩽ i ∧ i ⩽ -1 + # T ∧ # T ⩾ 0*
• *x__1* ∈ ran *T*
$\wedge$ *y__1* ∈ ran *T*
$\wedge$ ran *T* ∈ ℙ *({m1} ∪ ({m1'} ∪ {m2}))*
$\wedge$ *(T i = m1 ∧ T (1 + i) = m1'*
∨ *T i = m1' ∧ T (1 + i) = m1*
∨ *T i = m1 ∧ T (1 + i) = m2*
∨ *T i = m2 ∧ T (1 + i) = m1))))*
$\wedge$ *(¬ x = y ∧ (x = m1 ∨ x = m2) ∧ (y = m1 ∨ y = m2)*

$\Rightarrow (\exists T\_\_0\colon \mathrm{seq}\ Manager$
$\bullet (\forall i\_\_0\colon \mathbb{N} \mid 1 \leqslant i\_\_0 \land i\_\_0 \leqslant -1 + \# T\_\_0 \land \# T\_\_0 \geqslant 0$
$\bullet x \in \mathrm{ran}\ T\_\_0$
$\land y \in \mathrm{ran}\ T\_\_0$
$\land \mathrm{ran}\ T\_\_0 \in \mathbb{P} (\{m1\} \cup \{m2\})$
$\land (T\_\_0\ i\_\_0 = m1 \land T\_\_0\ (1 + i\_\_0) = m2$
$\lor T\_\_0\ i\_\_0 = m2 \land T\_\_0\ (1 + i\_\_0) = m1))))$
$\land (\neg x\_\_0 = y\_\_0$
$\land (x\_\_0 = m1 \lor x\_\_0 = m1' \lor x\_\_0 = m2)$
$\land (y\_\_0 = m1 \lor y\_\_0 = m1' \lor y\_\_0 = m2)$
$\Rightarrow (\exists T\_\_1\colon \mathrm{seq}\ Manager$
$\bullet (\forall i\_\_1\colon \mathbb{N} \mid 1 \leqslant i\_\_1 \land i\_\_1 \leqslant -1 + \# T\_\_1 \land \# T\_\_1 \geqslant 0$
$\bullet x\_\_0 \in \mathrm{ran}\ T\_\_1$
$\land y\_\_0 \in \mathrm{ran}\ T\_\_1$
$\land \mathrm{ran}\ T\_\_1 \in \mathbb{P} (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\land (T\_\_1\ i\_\_1 = m1 \land T\_\_1\ (1 + i\_\_1) = m1'$
$\lor T\_\_1\ i\_\_1 = m1' \land T\_\_1\ (1 + i\_\_1) = m1$
$\lor T\_\_1\ i\_\_1 = m1 \land T\_\_1\ (1 + i\_\_1) = m2$
$\lor T\_\_1\ i\_\_1 = m2 \land T\_\_1\ (1 + i\_\_1) = m1))))$

cases

$latency\ (m1, m2) = 7$
$\land replicate\ m2 = m1'$
$\land latency\ (m2, m1) = 2$
$\land \neg m1 = m1'$
$\land \neg m2 = m1'$
$\land maxLoad = 5$
$\land m1\ .\ load = 2$
$\land maxLatency = 5$
$\land m2\ .\ load = 8$
$\Rightarrow \{m1\} \cup (\{m1\} \cup (\{m1'\} \cup \{m2\})) = \{m1\} \cup (\{m1'\} \cup \{m2\})$

apply *extensionality2 to predicate* $\{m1\} \cup (\{m1\} \cup (\{m1'\} \cup \{m2\})) = \{m1\} \cup (\{m1'\} \cup \{m2\})$

$latency\ (m1, m2) = 7$
$\land replicate\ m2 = m1'$
$\land latency\ (m2, m1) = 2$
$\land \neg m1 = m1'$
$\land \neg m2 = m1'$
$\land maxLoad = 5$
$\land m1\ .\ load = 2$

$\land maxLatency = 5$
$\land m2 . load = 8$
$\Rightarrow \{m1\} \cup (\{m1\} \cup (\{m1'\} \cup \{m2\})) \in \mathbb{P} (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\land \{m1\} \cup (\{m1'\} \cup \{m2\}) \in \mathbb{P} (\{m1\} \cup (\{m1\} \cup (\{m1'\} \cup \{m2\})))$

prove by reduce

$true$

next

$latency (m1, m2) = 7$
$\land replicate\ m2 = m1'$
$\land latency (m2, m1) = 2$
$\land \neg m1 = m1'$
$\land \neg m2 = m1'$
$\land maxLoad = 5$
$\land m1 . load = 2$
$\land maxLatency = 5$
$\land m2 . load = 8$
$\land \neg x\_\_1 = y\_\_1$
$\land (x\_\_1 = m1 \lor x\_\_1 = m1' \lor x\_\_1 = m2)$
$\land (y\_\_1 = m1 \lor y\_\_1 = m1' \lor y\_\_1 = m2)$
$\Rightarrow (\exists T: \mathrm{seq}\ Manager$
$\bullet (\forall i: \mathbb{N} \mid 1 \leqslant i \land i \leqslant -1 + \# T \land \# T \geqslant 0$
$\bullet x\_\_1 \in \mathrm{ran}\ T$
$\land y\_\_1 \in \mathrm{ran}\ T$
$\land \mathrm{ran}\ T \in \mathbb{P} (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\land (T\ i = m1 \land T (1 + i) = m1'$
$\lor T\ i = m1' \land T (1 + i) = m1$
$\lor T\ i = m1 \land T (1 + i) = m2$
$\lor T\ i = m2 \land T (1 + i) = m1)))$

instantiate $T == \langle m1', m1, m2 \rangle$

$latency (m1, m2) = 7$
$\land replicate\ m2 = m1'$
$\land latency (m2, m1) = 2$
$\land \neg m1 = m1'$
$\land \neg m2 = m1'$
$\land maxLoad = 5$
$\land m1 . load = 2$
$\land maxLatency = 5$
$\land m2 . load = 8$
$\land \neg x = y$
$\land (x = m1 \lor x = m1' \lor x = m2)$
$\land (y = m1 \lor y = m1' \lor y = m2)$

$\wedge \neg (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \in \mathrm{seq}\ Manager$
$\wedge (\forall i: \mathbb{N}$
$\mid 1 \leqslant i$
$\wedge i \leqslant -1 + \# (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge \# (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \geqslant 0$
$\bullet (x \in \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge y \in \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \in \mathbb{P}\ (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\wedge (((\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1$
$\wedge (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1'$
$\vee (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1'$
$\wedge (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1$
$\vee (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1$
$\wedge (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m2$
$\vee (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m2$
$\wedge (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1))))$
$\Rightarrow (\exists T: \mathrm{seq}\ Manager$
$\bullet (\forall i\_\_0: \mathbb{N} \mid 1 \leqslant i\_\_0 \wedge i\_\_0 \leqslant -1 + \# T \wedge \# T \geqslant 0$
$\bullet x \in \mathrm{ran}\ T$
$\wedge y \in \mathrm{ran}\ T$
$\wedge \mathrm{ran}\ T \in \mathbb{P}\ (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\wedge (T\ i\_\_0 = m1 \wedge T\ (1 + i\_\_0) = m1'$
$\vee T\ i\_\_0 = m1' \wedge T\ (1 + i\_\_0) = m1$
$\vee T\ i\_\_0 = m1 \wedge T\ (1 + i\_\_0) = m2$
$\vee T\ i\_\_0 = m2 \wedge T\ (1 + i\_\_0) = m1))$

prove by reduce

true

next

$latency\ (m1, m2) = 7$
$\wedge replicate\ m2 = m1'$
$\wedge latency\ (m2, m1) = 2$
$\wedge \neg m1 = m1'$
$\wedge \neg m2 = m1'$
$\wedge maxLoad = 5$
$\wedge m1 . load = 2$
$\wedge maxLatency = 5$
$\wedge m2 . load = 8$
$\wedge \neg x = y$
$\wedge (x = m1 \vee x = m2)$
$\wedge (y = m1 \vee y = m2)$
$\Rightarrow (\exists T\_\_0: \mathrm{seq}\ Manager$

$\bullet\ (\forall i\_\_0: \mathbb{N} \mid 1 \leqslant i\_\_0 \wedge i\_\_0 \leqslant -1 + \# T\_\_0 \wedge \# T\_\_0 \geqslant 0$
$\bullet\ x \in \mathrm{ran}\ T\_\_0$
$\wedge\ y \in \mathrm{ran}\ T\_\_0$
$\wedge\ \mathrm{ran}\ T\_\_0 \in \mathbb{P}\ (\{m1\} \cup \{m2\})$
$\wedge\ (T\_\_0\ i\_\_0 = m1 \wedge T\_\_0\ (1 + i\_\_0) = m2$
$\vee\ T\_\_0\ i\_\_0 = m2 \wedge T\_\_0\ (1 + i\_\_0) = m1)))$

instantiate $T\_\_0 == \langle m1', m1, m2 \rangle$

$latency\ (m1, m2) = 7$
$\wedge\ replicate\ m2 = m1'$
$\wedge\ latency\ (m2, m1) = 2$
$\wedge\ \neg\ m1 = m1'$
$\wedge\ \neg\ m2 = m1'$
$\wedge\ maxLoad = 5$
$\wedge\ m1\ .\ load = 2$
$\wedge\ maxLatency = 5$
$\wedge\ m2\ .\ load = 8$
$\wedge\ \neg\ x = y$
$\wedge\ (x = m1 \vee x = m2)$
$\wedge\ (y = m1 \vee y = m2)$
$\wedge\ \neg\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle) \in \mathrm{seq}\ Manager$
$\wedge\ (\forall i: \mathbb{N}$
$\mid 1 \leqslant i$
$\wedge\ i \leqslant -1 + \#\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge\ \#\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \geqslant 0$
$\bullet\ (x \in \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge\ y \in \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge\ \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \in \mathbb{P}\ (\{m1\} \cup \{m2\})$
$\wedge\ (((\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1$
$\wedge\ ((\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m2$
$\vee\ ((\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m2$
$\wedge\ ((\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1))))$
$\Rightarrow (\exists T: \mathrm{seq}\ Manager$
$\bullet\ (\forall i\_\_0: \mathbb{N} \mid 1 \leqslant i\_\_0 \wedge i\_\_0 \leqslant -1 + \# T \wedge \# T \geqslant 0$
$\bullet\ x \in \mathrm{ran}\ T$
$\wedge\ y \in \mathrm{ran}\ T$
$\wedge\ \mathrm{ran}\ T \in \mathbb{P}\ (\{m1\} \cup \{m2\})$
$\wedge\ (T\ i\_\_0 = m1 \wedge T\ (1 + i\_\_0) = m2$
$\vee\ T\ i\_\_0 = m2 \wedge T\ (1 + i\_\_0) = m1)))$

prove by reduce

$$
\begin{aligned}
& i \in \mathbb{Z} \\
& \land\ latency\ (m1, m2) = 7 \\
& \land\ replicate\ m2 = m1' \\
& \land\ latency\ (m2, m1) = 2 \\
& \land\ \neg\ m1 = m1' \\
& \land\ \neg\ m2 = m1' \\
& \land\ maxLoad = 5 \\
& \land\ m1\ .\ load = 2 \\
& \land\ maxLatency = 5 \\
& \land\ m2\ .\ load = 8 \\
& \land\ \neg\ x = y \\
& \land\ i \geqslant 0 \\
& \land\ 1 \leqslant i \\
& \land\ i \leqslant 2 \\
& \land\ (x = m1 \lor x = m2) \\
& \land\ (y = m1 \lor y = m2) \\
& \Rightarrow (\exists T: \mathrm{seq}\ Manager \\
& \bullet\ (\forall i\_\_0: \mathbb{N} \mid 1 \leqslant i\_\_0 \land i\_\_0 \leqslant -1 + \#\ T \land \#\ T \geqslant 0 \\
& \bullet\ x \in \mathrm{ran}\ T \\
& \land\ y \in \mathrm{ran}\ T \\
& \land\ \mathrm{ran}\ T \in \mathbb{P}\ (\{m1\} \cup \{m2\}) \\
& \land\ (T\ i\_\_0 = m1 \land T\ (1 + i\_\_0) = m2 \\
& \lor\ T\ i\_\_0 = m2 \land T\ (1 + i\_\_0) = m1)))
\end{aligned}
$$

instantiate $T == \langle m2, m1 \rangle$

$$
\begin{aligned}
& i \in \mathbb{Z} \\
& \land\ latency\ (m1, m2) = 7 \\
& \land\ replicate\ m2 = m1' \\
& \land\ latency\ (m2, m1) = 2 \\
& \land\ \neg\ m1 = m1' \\
& \land\ \neg\ m2 = m1' \\
& \land\ maxLoad = 5 \\
& \land\ m1\ .\ load = 2 \\
& \land\ maxLatency = 5 \\
& \land\ m2\ .\ load = 8 \\
& \land\ \neg\ x = y \\
& \land\ i \geqslant 0 \\
& \land\ 1 \leqslant i
\end{aligned}
$$

$\wedge\ i \leqslant 2$
$\wedge\ (x = m1 \vee x = m2)$
$\wedge\ (y = m1 \vee y = m2)$
$\wedge\ \neg\ (\langle m2\rangle \frown \langle m1\rangle \in \mathrm{seq}\ Manager$
$\wedge\ (\forall i\_\_0: \mathbb{N}$
$\mid 1 \leqslant i\_\_0 \wedge i\_\_0 \leqslant -1 + \#\ (\langle m2\rangle \frown \langle m1\rangle) \wedge \#\ (\langle m2\rangle \frown \langle m1\rangle) \geqslant 0$
$\bullet\ (x \in \mathrm{ran}\ (\langle m2\rangle \frown \langle m1\rangle)$
$\wedge\ y \in \mathrm{ran}\ (\langle m2\rangle \frown \langle m1\rangle)$
$\wedge\ \mathrm{ran}\ (\langle m2\rangle \frown \langle m1\rangle) \in \mathbb{P}\ (\{m1\} \cup \{m2\})$
$\wedge\ ((\langle m2\rangle \frown \langle m1\rangle)\ i\_\_0 = m1 \wedge (\langle m2\rangle \frown \langle m1\rangle)\ (1 + i\_\_0) = m2$
$\vee\ (\langle m2\rangle \frown \langle m1\rangle)\ i\_\_0 = m2 \wedge (\langle m2\rangle \frown \langle m1\rangle)\ (1 + i\_\_0) = m1))))$
$\Rightarrow (\exists T: \mathrm{seq}\ Manager$
$\bullet\ (\forall i\_\_1: \mathbb{N} \mid 1 \leqslant i\_\_1 \wedge i\_\_1 \leqslant -1 + \#\ T \wedge \#\ T \geqslant 0$
$\bullet\ x \in \mathrm{ran}\ T$
$\wedge\ y \in \mathrm{ran}\ T$
$\wedge\ \mathrm{ran}\ T \in \mathbb{P}\ (\{m1\} \cup \{m2\})$
$\wedge\ (T\ i\_\_1 = m1 \wedge T\ (1 + i\_\_1) = m2$
$\vee\ T\ i\_\_1 = m2 \wedge T\ (1 + i\_\_1) = m1)))$

prove by reduce

true

next

$latency\ (m1, m2) = 7$
$\wedge\ replicate\ m2 = m1'$
$\wedge\ latency\ (m2, m1) = 2$
$\wedge\ \neg\ m1 = m1'$
$\wedge\ \neg\ m2 = m1'$
$\wedge\ maxLoad = 5$
$\wedge\ m1\ .\ load = 2$
$\wedge\ maxLatency = 5$
$\wedge\ m2\ .\ load = 8$
$\wedge\ \neg\ x\_\_0 = y\_\_0$
$\wedge\ (x\_\_0 = m1 \vee x\_\_0 = m1' \vee x\_\_0 = m2)$
$\wedge\ (y\_\_0 = m1 \vee y\_\_0 = m1' \vee y\_\_0 = m2)$
$\Rightarrow (\exists T\_\_1: \mathrm{seq}\ Manager$
$\bullet\ (\forall i\_\_1: \mathbb{N} \mid 1 \leqslant i\_\_1 \wedge i\_\_1 \leqslant -1 + \#\ T\_\_1 \wedge \#\ T\_\_1 \geqslant 0$
$\bullet\ x\_\_0 \in \mathrm{ran}\ T\_\_1$
$\wedge\ y\_\_0 \in \mathrm{ran}\ T\_\_1$
$\wedge\ \mathrm{ran}\ T\_\_1 \in \mathbb{P}\ (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\wedge\ (T\_\_1\ i\_\_1 = m1 \wedge T\_\_1\ (1 + i\_\_1) = m1'$

$\vee\ T\_\_1\ i\_\_1 = m1' \wedge T\_\_1\ (1 + i\_\_1) = m1$
$\vee\ T\_\_1\ i\_\_1 = m1 \wedge T\_\_1\ (1 + i\_\_1) = m2$
$\vee\ T\_\_1\ i\_\_1 = m2 \wedge T\_\_1\ (1 + i\_\_1) = m1)))$

instantiate $T\_\_1 == \langle m1', m1, m2 \rangle$

$latency\ (m1, m2) = 7$
$\wedge\ replicate\ m2 = m1'$
$\wedge\ latency\ (m2, m1) = 2$
$\wedge\ \neg\ m1 = m1'$
$\wedge\ \neg\ m2 = m1'$
$\wedge\ maxLoad = 5$
$\wedge\ m1\ .\ load = 2$
$\wedge\ maxLatency = 5$
$\wedge\ m2\ .\ load = 8$
$\wedge\ \neg\ x = y$
$\wedge\ (x = m1 \vee x = m1' \vee x = m2)$
$\wedge\ (y = m1 \vee y = m1' \vee y = m2)$
$\wedge\ \neg\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle) \in \mathrm{seq}\ Manager$
$\wedge\ (\forall i: \mathbb{N}$
$|\ 1 \leqslant i$
$\wedge\ i \leqslant -1 + \#\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge\ \#\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \geqslant 0$
$\bullet\ (x \in \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge\ y \in \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))$
$\wedge\ \mathrm{ran}\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle)) \in \mathbb{P}\ (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\wedge\ (((\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1$
$\wedge\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1'$
$\vee\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1'$
$\wedge\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1$
$\vee\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m1$
$\wedge\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m2$
$\vee\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ i = m2$
$\wedge\ (\langle m1' \rangle \frown (\langle m1 \rangle \frown \langle m2 \rangle))\ (1 + i) = m1))))$
$\Rightarrow (\exists T: \mathrm{seq}\ Manager$
$\bullet\ (\forall i\_\_0: \mathbb{N}\ |\ 1 \leqslant i\_\_0 \wedge i\_\_0 \leqslant -1 + \#\ T \wedge \#\ T \geqslant 0$
$\bullet\ x \in \mathrm{ran}\ T$
$\wedge\ y \in \mathrm{ran}\ T$
$\wedge\ \mathrm{ran}\ T \in \mathbb{P}\ (\{m1\} \cup (\{m1'\} \cup \{m2\}))$
$\wedge\ (T\ i\_\_0 = m1 \wedge T\ (1 + i\_\_0) = m1'$

$\vee$ *T i__0 = m1' ∧ T (1 + i__0) = m1*
$\vee$ *T i__0 = m1 ∧ T (1 + i__0) = m2*

$\vee$ *T i__0 = m2 ∧ T (1 + i__0) = m1)))*

prove by reduce

true

next

true