DC Sec

Leibniz Universität Hannover

Homomorphic Cryptography for Cloud Computing

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Outline

Real World Applications

What is Homomorphic Cryptography?

Homomorphism := **Structure-preserving map**

w.r.t. operations:

$$\varphi(a) + \varphi(b) = \varphi(a + b)$$

$$\varphi(a) \cdot \varphi(b) = \varphi(a \cdot b)$$

Crypto scheme:

$$\varphi \equiv \text{Encrypt}$$

$$\varphi^{-1} \equiv \text{Decrypt}$$

Conclusions:

- Evaluation of arbitrary formulas with $+$ and $\cdot$
- Decryption yields sum or product

$\Rightarrow$ **We can do stuff with ciphertexts!**

What All Can We Do?

$\Rightarrow$ **We can do stuff with ciphertexts!**

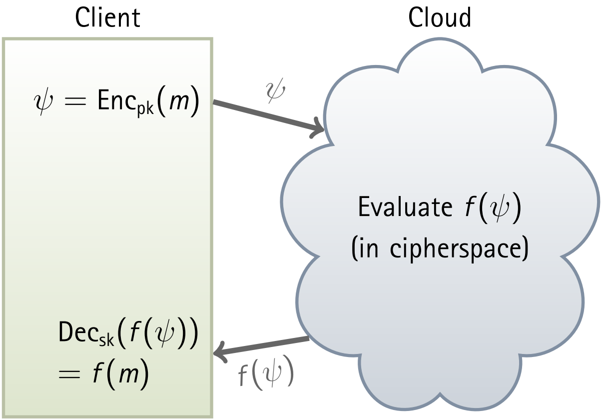
Assume a homomorphic encryption scheme. What do we get?

- Homomorphism: Addition/Multiplication of ciphertexts yields sum/product after decryption
- Next step: Plaintexts are Bits ($\mathcal{P} = \mathbb{Z}/2\mathbb{Z}$)
- Then:

$$\left.\begin{array}{llll} a+b & \bmod 2 \rightsquigarrow a \oplus b & \text{(xor)} \\ a \cdot b & \bmod 2 \rightsquigarrow a \wedge b & \text{(and)} \end{array}\right\} \rightsquigarrow \text{Arbitrary boolean circuits}$$

- Finally: Build a CPU out of boolean circuits
  $\rightsquigarrow$ Arbitrary programs in cipherspace.

Use-case in Cloud Computing



Notation:

m	plaintext
ψ	ciphertext
f	program
pk	public key
sk	secret key
Enc_{pk}	encryption
Dec_{sk}	decryption

History of Homomorphic Cryptography

"Holy Grail" of cryptography for a long time

1978 Posed as open problem by Rivest *et al.*

2005 Evaluate 2-DNF formulas on ciphertexts, Boneh *et al.*

2009 Fully homomorphic encryption using ideal lattices by Craig Gentry

2010 Fully homomorphic encryption over the integers by van Dijk *et al.*

2012 Fully homomorphic encryption without bootstrapping by Brakerski *et al.*

Gentry's original scheme

Common construction:

- Plaintext, ciphertext are rings (operations $+$ and $\cdot$)
- Encryption is a homomorphism from plaintext to ciphertext
- Operations on ciphertexts add noise
- Decryption succeeds as long as noise remains within bounds

"Cleaning" the ciphertext (*bootstrapping*):

- Represent decryption function as boolean circuit
- Decrypt a ciphertext in ciphertext space $\rightsquigarrow$ "cleaner" new ciphertext
- Requirements:
  - Decryption circuit must be shallow enough
  - Called *bootstrappable*-property

A simple homomorphic scheme by example

A simple homomorphic scheme by example

Goal:

- Simple, easy to understand homomorphic scheme
- Symmetric scheme (with key p)
- Hardness based on prime number factorization
- Not bootstrappable (i.e. not *fully* homomorphic)

Notation:

- $\mathbb{P}$ – number of primes
- $a \overset{R}{\leftarrow} A$ – choose a from set A with uniform distribution

Keygen, Encrypt, Decrypt

$p \leftarrow \text{Keygen}(\lambda)$

1: **return** random λ-bit prime

$c \leftarrow \text{Encrypt}_\lambda(m, p)$

1: $r \xleftarrow{R} \mathbb{N}$
2: $q \xleftarrow{R}$ random λ-bit number
3: **return** $m + 2r + pq$

$m \leftarrow \text{Decrypt}(c, p)$

1: **return** $(c \mod p) \mod 2$

Correctness:

$$\begin{aligned} & \text{Dec}(\text{Enc}(m, p)) \stackrel{!}{=} m \\ \Leftrightarrow\ & ((m + 2r + pq) \mod p) \mod 2 \\ =\ & (m + 2r) \mod 2 = m \end{aligned}$$

Bit Operations

Xor(c_1, c_2)

1: **return** $c_1 + c_2$

And(c_1, c_2)

1: **return** $c_1 \cdot c_2$

Correctness

Remember: $\text{Encrypt}(m, p) := c \leftarrow m + 2r + pq$

Xor / Addition

$$\text{Decrypt}(\text{Xor}(c_1, c_2), p) = \text{Decrypt}(m_1 + 2r_1 + pq_1 + m_2 + 2r_2 + pq_2, p) =$$
$$\left(((m_1 + m_2) + 2(r_1 + r_2) + p(q_1 + q_2)) \mod p\right) \mod 2 = m_1 + m_2$$

And / Multiplication

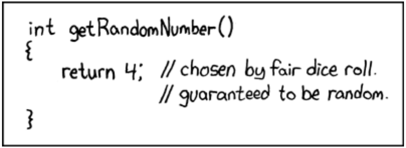
$$\text{Decrypt}(\text{And}(c_1, c_2), p) =$$
$$\text{Decrypt}((m_1 + 2r_1 + pq_1)(m_2 + 2r_2 + pq_2), p) =$$
$$\left((m_1 \cdot m_2 + 2r_2 m_1 + pq_2 m_1 + 2r_1 c_2 + pq_1 c_2) \mod p\right) \mod 2 = m_1 \cdot m_2$$

Putting in Numbers

- $p \leftarrow \text{Keygen}()$: Choose $p = 23$
- $c_0 \leftarrow \text{Encrypt}(0, p)$:
  - Choose $q \leftarrow 5$
  - Choose $r \leftarrow 3$
  - Choose

    $c_0 \leftarrow 0 + 2 \cdot 3 + 5 \cdot 23 = 121$
- $c_1 \leftarrow \text{Encrypt}(1, p)$:
  - Choose $q \leftarrow 4$
  - Choose $r \leftarrow 4$
  - Choose

    $c_1 \leftarrow 1 + 4 \cdot 3 + 4 \cdot 23 = 101$



Examples

$c_0 \oplus c_1 = 121 + 101 = 222$

$222 \mod 23 = 15 \rightsquigarrow 1$

$c_0 \wedge c_1 = 121 \cdot 101 = 12221$

$12221 \mod 23 = 8 \rightsquigarrow 0$

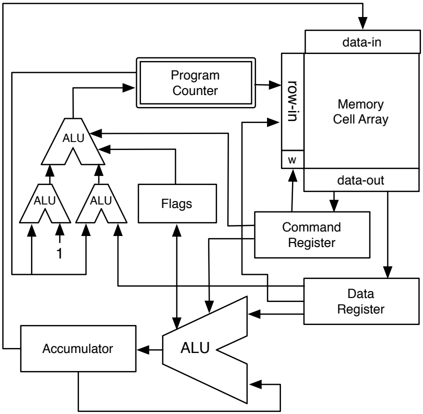
The Problem With The Noise

Remember: $m \leftarrow \text{Decrypt}(c) = (c \mod p) \mod 2$

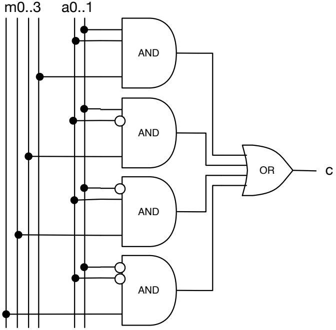
Decryption only works iff. $a + b < p$, $a \cdot b < p$

	$p \cdot q_1$	$2 \cdot a_1$	m_1
$\oplus$	$p \cdot q_2$	$2 \cdot a_2$	m_2
$=$	$p \cdot (q_1 + q_2)$	$2 \cdot (a_1 + a_2)$	$m_1 + m_2$

Encrypted CPU — Overview



Encrypted memory access — reading



m single-bit memory cells

a address

c selected cell

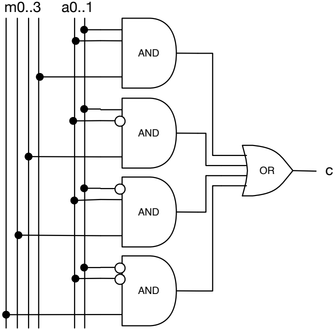
Selection circuit

$$c(a, m) =$$
$$(\neg a_0 \wedge \neg a_1 \wedge m_0) \oplus (a_0 \wedge \neg a_1 \wedge m_1) \oplus$$
$$(\neg a_0 \wedge a_1 \wedge m_2) \oplus (a_0 \wedge a_1 \wedge m_3)$$

Analysis

- Two memory addresses indistinguishable
- Access pattern hidden
- $\Rightarrow$ *Oblivious read access*

Encrypted memory access — writing



m single-bit memory cells

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c selected cell

Selection circuit

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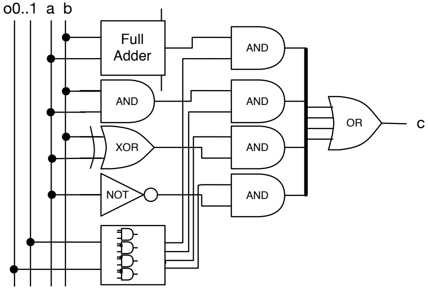
$$(\neg a_0 \wedge a_1 \wedge m_2) \oplus (a_0 \wedge a_1 \wedge m_3)$$

Write access (write d to address a)

- For each memory cell m:
  - $m \leftarrow (c(a, m) \wedge d) \oplus (\neg c(a, m) \oplus m)$
- $\Rightarrow$ *Oblivious write access*

Encrypted Arithmetic–Logical Unit



o opcode

a first parameter

b second parameter

Opcodes

o_0	o_1	Output	o_0	o_1	Output
0	0	$a + b$	1	0	$a \oplus b$
0	1	$a \wedge b$	1	1	$\neg a$

- ALU function selection same as memory selection
- From here:
  - Fix machine word (i.e. 8 bits)
  - Add circuits for full adder, carry-flag, zero-flag etc.

Plugging it together

- ALU for arithmetic and logic operations
- Group of smaller ALUs for program flow control etc.
- Registers: encrypted bit columns
- Memory: memory cells (encrypted bit columns) with access logic
- Simple processor cycle:

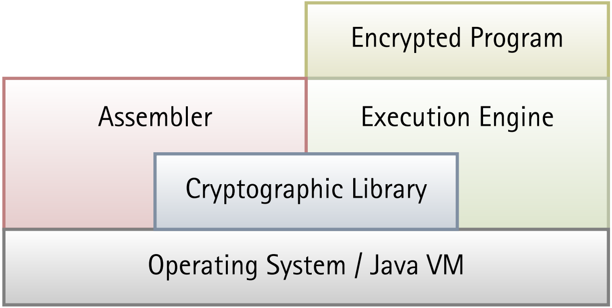
  FETCH1 read memory cell pointed at by program counter

  FETCH2 read memory cell pointed at by fetched operand

  EXEC execute operation in command register

  WRITE write results to memory
- *Note:* Every cycle performs all three memory access operations (needed for obliviousness)

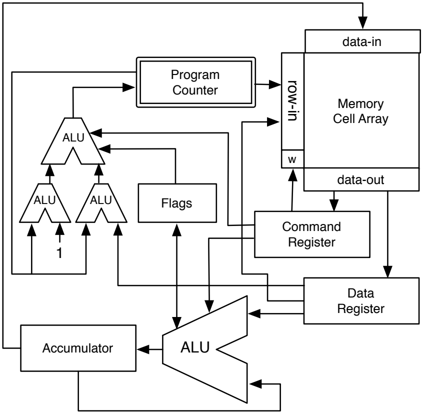
System architecture



Implementation Details

- Memory word length: 13 bits = 8 bits data + 5 bits opcode
- Architecture independent from concrete cryptosystem
- One cycle takes $\approx$ 2 ms (2.4 GHz Intel Core 2 Duo)

Encrypted CPU — Summary



- Data is encrypted
  - Read accessible
  - Write accessible
- Program is encrypted
  - All opcodes are encrypted
  - ALU output is encrypted

Code at http://hcrypt.com/shape-cpu/

Encrypted, Turing–complete machine

Real World Applications

1 Introduction

2 A simple homomorphic scheme by example

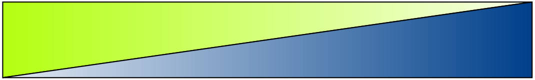
3 An Encrypted CPU for Homomorphic Cryptography
- Encrypted memory access – reading
- Encrypted memory access – writing
- Encrypted Arithmetic-Logical Unit

4 Real World Applications
- Why are we not done yet?
- Searching on Encrypted Data

5 Conclusion

Why are we not done yet?

Performance



Confidentiality

unencrypted hybrid algorithm encrypted CPU

Unencrypted: very fast, familiar tools (compiler etc.)

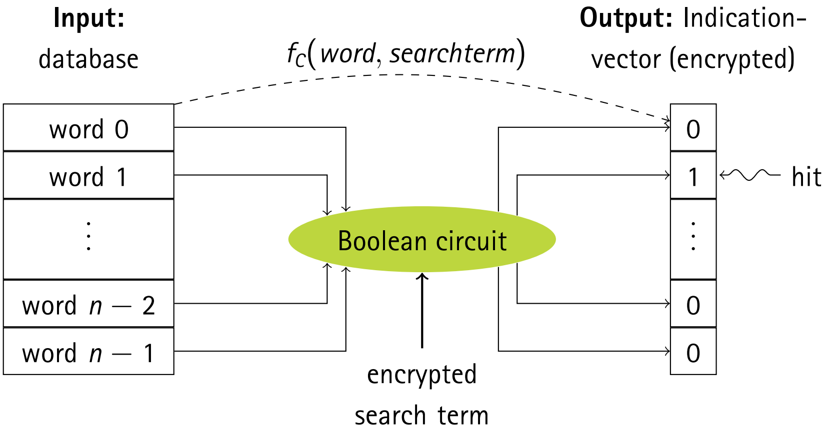
but requires trust

Encrypted CPU: Turing-complete, encrypts data and program

but bad performance

Hybrid algorithm: seeks a compromise: protects confidential parts (just data, just part of the data etc.), performs better than encrypted CPU

Search with Boolean circuits



Exact Search

Boolean circuit f_c: Conjunction over single letters

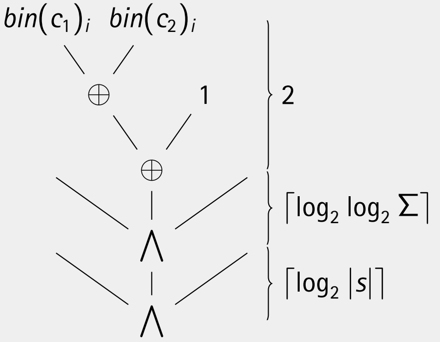
$$\bigwedge_{i=0}^{|s|-1} (w_i =_c s_i)$$

Comparison $=_c$: Conjunction over binary representation

$$\bigwedge_{i=0}^{\lceil \log_2 \Sigma \rceil - 1} bin(c_1)_i \oplus bin(c_2)_i \oplus 1$$

$a \oplus b \oplus 1$	0	1
0	1	0
1	0	1

Depth of f_c:



Use Case: Search on Human Genomes

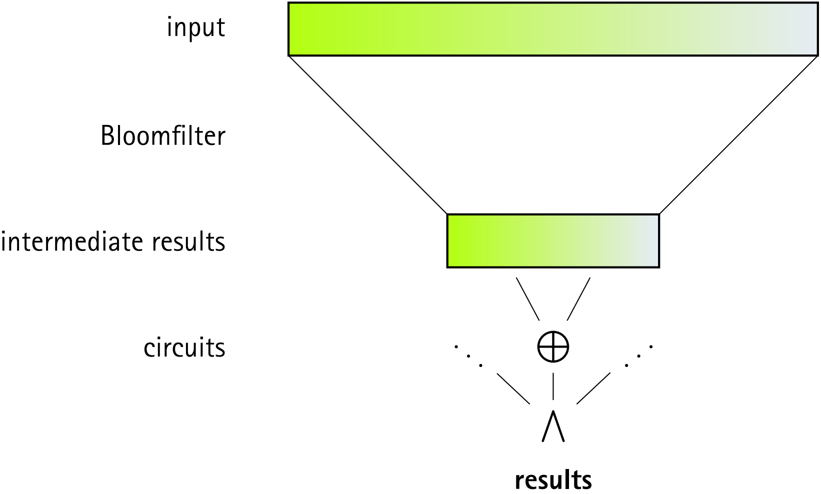
Motivation:

- Large database of human genomes
- Future: personalized medications and therapies
- Depend on patients' DNA
- $\Rightarrow$ database is public, query is confidential

But ...

- too much data (100 MB up to several GB)
- kills performance even with customized circuit
- Solution: Prefilter results $\rightsquigarrow$ Bloomfilter Search
- $\rightsquigarrow$ Hybrid algorithms

Hybrid Algorithm for Searching



Bloomfilters for words and sets

n hash functions $f_0 \dots f_{n-1}$; Bloomfilter length m; alphabet Σ; word $w \in \Sigma$

helper function $b(f(w)) = (\dots, 0, \underbrace{1}_{\text{at } f(w)}, 0, \dots) \in [0,1]^m$

Definition of $\mathcal{B}(w)$:

$$\mathcal{B} : \begin{array}{l} \Sigma^* \to [0,n]^m \\ w \mapsto \sum_{i=1}^{n} b(f_i(w)) \end{array}$$

$f_1(w) = 2$ $\quad$ $f_0(w) = 7$

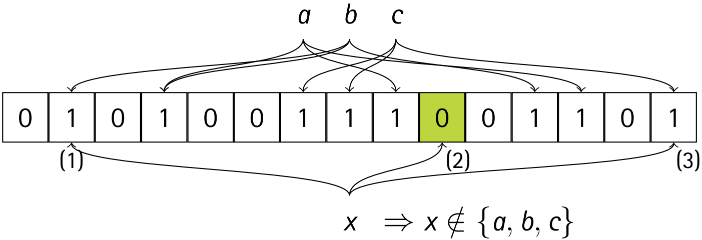
0	1	2	3	4	5	6	7	8
0	0	1	0	0	0	0	1	0

Definition of $\widehat{\mathcal{B}}(M)$:

$$\widehat{\mathcal{B}} : \begin{array}{l} \wp(\Sigma^*) \to [0,n]^m \\ M \mapsto \sum_{w \in M} \mathcal{B}(w) \end{array}$$

	0	1	2	3	4	5	6	7	8
w_0	0	0	1	0	0	0	0	1	0
w_1	1	0	0	0	0	1	0	0	0
$\{w_0, w_1\}$	1	0	1	0	0	1	0	1	0

Set membership with Bloomfilters



Bloomfilter with three hash functions

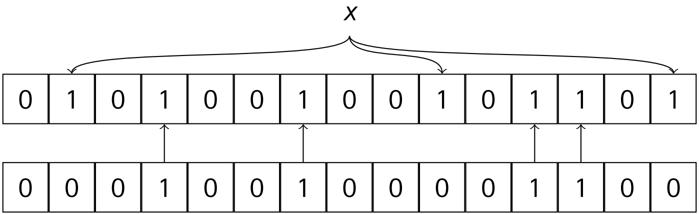
$$\mathcal{B}(w) \notin \widehat{\mathcal{B}}(M) :\Leftrightarrow \exists i \in \text{index set} : \mathcal{B}(w)[i] > \widehat{\mathcal{B}}(M)[i] \Rightarrow w \notin M$$

$$\mathcal{B}(w) \in \widehat{\mathcal{B}}(M) :\Leftrightarrow \forall i \in \text{index set} : \mathcal{B}(w)[i] \leq \widehat{\mathcal{B}}(M)[i] \Rightarrow w \in M$$

Obfuscation of Bloomfilters

Goal: Fuzzyness in set membership: make $w \in M$ more probable

Method: Obfuscation with parameter λ (here $\lambda = 4$)

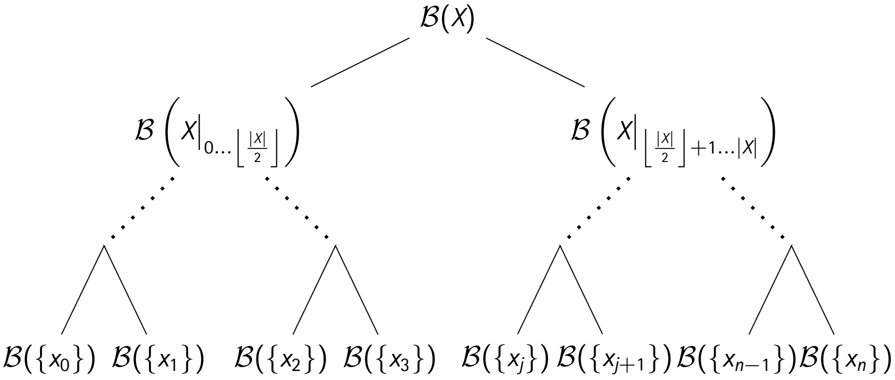


Result: Given a Bloomfilter $\mathbf{B} = \mathcal{B}^{k,m}(A)$ for a set A and a (plain) Bloom filter $\mathbf{b} = \mathcal{B}^{k,m}(a)$ as well as an obfuscated version $\mathbf{b}' = \text{obfuscate}(\mathbf{b}, m, \lambda)$:

- $\mathbf{b} \in \mathbf{B} \Rightarrow \mathbf{b}' \in \mathbf{B}$
- $\mathbf{b}' \in \mathbf{B} \Rightarrow \Pr[\mathbf{b} \in \mathbf{B}] = 1/\binom{k+\lambda}{k} \rightsquigarrow$ hiding in x %

Index search with Bloomfilters

- Bloomfilter tree: Index for database X
- Search: divide-and-conquer in $\mathcal{O}(\log(X))$



Conclusion

- Homomorphic cryptography is a powerful tool
- Relatively new development, more to come
- Promising new applications, especially with security / performance tradeoffs
- Code available at `http://hcrypt.com`

Further research

- Faster homomorphic cryptoschemes
- More hybrid algorithms $\rightsquigarrow$ more applications

Thank You!